# Conical Travel Time Functions for the GTA Road Network 

by

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A project submitted in conformity with the requirements for the degree of Master of Engineering Graduate Department of Civil Engineering

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#### Abstract

Conical Travel Time Functions for the GTA Road Network M.Eng., 1999

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A consistent series of conical volume travel time functions capable of representing travel times observed under specific road and traffic conditions were developed. The identification of particular travel time characteristics was possible with the analysis of a previously collected database containing substantial information on arterial travel time for signalized road sections around the Greater Toronto Area (GTA). The road parameters of presence of transit vehicles, speed limit, signal frequency and headway between buses were used to determine particular road conditions used to form groups of empirical data with specific travel time characteristics. A combination of judgment and curve fitting was used to develop a set of conical curves to imitate the empirical volume travel time relationships on the specific groups. The mathematical properties of the conical formulation should provide significant computational efficiencies on the user equilibrium assignment process. The application of these functions could generate potential improvements in modelling procedures for the GTA, in that simulation travel times would be a better representation of actual travel times.


Acknowledgments

Professor G. N. Steuart; Thanks for your patience, support and wise instruction.

Thanks to my Family.

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## INTRODUCTION

The project presents a consistent series of conical volume travel time functions for the Greater Toronto Area (GTA). These calibrated volume travel time functions are proposed to be applied in the existing modelling procedures used to simulate the urban transportation system of the GTA. The functions provide an accurate representation of empirical travel times under specific road conditions and could improve the accuracy of the traffic assignment process. It is anticipated that the mathematical properties of the conical formulation will produce a faster convergence in the user equilibrium assignment process.

The functions were developed with information collected in a previous empirical study of arterial travel times for signalized road sections on the GTA. The first objective in this analysis of these data was to identify significant road conditions influencing the observed travel times. The presence of streetcars, the speed limit, the number of signalized intersections per kilometer and the headway between buses were selected as significant road parameters influencing empirical travel times. Special values for these selected road parameters were used to create different categories, which were used to group the information in such a way that the number of empirical observations in the formed groups will be sufficient to reflect their influence on travel time in each of the groups. The second objective was to calibrate the parameters of the conical volume delay function in order to fit the conical curves to the particular volume travel time relationships observed for the specific groups formed.

Strategic transportation modelling in recent times has become an important element in effective transportation planning. The high capacities of computing resources at low cost have permit transportation modelling to play an important role in effective transportation planning. Transportation problems such as congestion, pollution, accidents and financial deficits in the 1960s and 1970s clearly reflect a need to improve the transportation planning process and have encouraged the development of transportation modelling procedures.

Transportation planning is necessary to provide information to a decision making process for policy issues and capital works programs. Transportation services touch on almost all social and classes in a city and, therefore, represent a key issue for policy making. Throughout history the urban transportation systems has proven to be a critical factor for economic, social and urban development.

Regional Municipalities and different agencies in the GTA are aware of the benefits that urban transportation modelling techniques can produce and they have undertaken joint efforts to develop an electronic representation of the road and transit system. This electronic network determines vehicular and person travel paths and is needed for the trip assignment stage of a typical travel demand modelling process. The network is developed on the EMME/2 system that provides a general framework to implement a wide variety of travel demand forecasting procedures.

The volume travel time functions are a critical element in this electronic representation of the transportation system. The volume travel time functions are entered as attributes of links, which in turn represent road sections. The purpose of these volume travel time functions is to determine the travel time according to the volume on each network link. To allocate trips to the network, the user equilibrium assignment technique is used. The behavioral assumption of the equilibrium assignment problem is that each user chooses the route that he perceives to be the best (shortest travel time). The resulting flows on every link should satisfy Wardrop's (1952) user optimal principle, that no user can improve his travel time by changing routes. The consequence is that the equilibrium traffic assignment corresponds to a set of flows such that all paths used between an origin-destination pair are of equal time. It can be observed that the assignment process depends on the travel time for the user on specific paths. This travel time is given by the volume travel time functions and, therefore, the accuracy of the volume travel time functions is critical to the results of the model.

A review of the ability of the traffic assignment process to represent real travel times and link volumes on the GTA network is needed. Network characteristics in the traffic assignment process in the GTA demand models have changed very little since the mid 1970's. The development of electronic networks for the City of Toronto started in the 1960's with the combined efforts of the Ministry of Transportation of Ontario (MTO) and Metropolitan Toronto. The first network was developed with UTMS, a widely used system in North America. A software package called System 33 was subsequently introduced as an evolutionary step in road network representation. Neither system provided a geographic
base and all the information required was stored on link attributes. The allocation of trips to these networks was carried out using various capacity restraint techniques rather than user equilibrium. The representation of travel time on the networks was done with the BPR volume delay functions. Around 1989 the EMME/2 system was used to update the network and the assignment technique, this system provided a geographic base to the network but the volume delay functions did not change.

## Michael Mahut Study

In an attempt to improve the accuracy of the traffic assignment techniques used in the GTA model, Michael Mahut (1990) identified the relevance of the volume travel time functions as a key element in the process and conducted an empirical study of arterial travel time in the GTA area. The main concern of the study was to verify the accuracy of the volume travel time functions used to represent real travel time and also to identify the traffic and road parameters that affect the observed travel times.

The study was called "A Parametric Analysis of Arterial Travel Time", and conducted an extensive data collection effort on travel times on arterial roads in the GTA. Data was collected over a wide range of traffic flow conditions as well as road parameters such as speed limit, signal frequency, lane width, land use and the presence of transit vehicles. The method used to measure travel time and flow in this study was the moving observer method developed by the British Road Research Laboratory Traffic and Safety Division.

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The survey area consisted of four of the six regional municipalities within the Greater Toronto Area: Metropolitan Toronto, Peel, York and Durham. The data were gathered during the morning (6:45 to $9: 45$ ) and afternoon (15:30 to $18: 30$ ) peak periods on Tuesdays, Wednesdays and Thursdays, between May 2 and August 17, 1995. In total, 492 bi-directional road sections were selected with three to four runs on each direction for a total of 3472 data points.

Mahut used the empirical observations to conduct a preliminary analysis of arterial travel times. The analysis tried to identify the effect of flow and road parameters on travel time. To analyze travel time, the study broke down the total journey time into link time and intersection delay. Link time was defined as total travel time minus signalized intersection delay. He recognized that the presence of streetcars on a road section completely altered the behavior of the travel time on a link. Therefore, he divided the data in two groups called "streetcars" and "no streetcars". On these two groups he used the speed limit on the link to create more divisions on the database. Five particular speed limit groups were created. The information was grouped in a particular way where at least these two road parameters were held constant. This action improved the conditions for further analysis, isolating cause and effect. The rest of the analysis focused on finding relationships for travel, link and intersection delay times with flow and signal frequency (per km ) on each of the different groups.

Some of the main findings of Mahut's study show that for roads without streetcars, travel time appeared to be more strongly correlated with signal frequency than with flow. Link
time was found to have a significant linear relationship with both flow and signal frequency for these roads, while intersection delay was found to be only correlated with the number of signals. On roads with streetcars, flow was not found to be useful in predicting travel time. Linear relationships for travel time and link time with signal frequency were found to be fairly strong, as was a relationship between intersection delay and the number of signals. Travel time functions were formulated to replicate empirical data.

## Literature Review for Volume Travel Time Functions

Branston (1975) in an extensive review of volume delay functions, isolated two main approaches used by researchers to define volume-delay functions, mathematical and theoretical. The theoretical functions may not necessarily lead to a simple functional relationship between travel time or journey speed and flow. They may require more information on network characteristics such as signal spacing, settings and street width to be input. These functions consider running time and queueing time separately. The mathematical functions usually guarantee a relatively simple relationship between travel time and flow and the parameters of the functions can be related in a "known way" to link characteristics.

Wardrop (1968) proposed a model based on running speed as a function of flow and road width, and delay time per unit distance as a function of flow, road width, and average affective green split. It was observed that the primary cause of delay with increasing flow
is the presence of intersections, then the effect of these intersections on journey time per unit length will depend on how large the delay time is relative to the running time. This in turn depends on the frequency of signalized intersections (per unit distance), and on the relationship between total intersection delay and intersection frequency, which one would expect to be influenced by efforts at signal synchronisation. A relationship based on the Webster function for delay at fixed time (1958) was incorporated in the model.

Irwin, Dodd and Von Cube (1961) proposed mathematical functions of link capacity to be used in assignment procedures. Their model consists of two straight line segments where the change in slope occurs at a value of flow they termed "critical capacity". This study used the signal frequency (per mile) as a parameter. Also Irwin and Von Cube (1962) used the presence of transit vehicles (in addition to signal frequency) as parameters. They made the assumption that the presence of transit vehicles would affect the volume delay relationship by reducing the capacity of a link. Both studies used empirical data of delay against flow collected at signalized intersections in Toronto as a basis for their formulations.

Many different functional forms for volume delay functions have been proposed and used in the past. One of the main limitations for developing new functions is data collection. As noticed by Michael Mahut, it has been common for data collected in one study to be used for several years by different researchers to calibrate new functional forms without collecting new observations. Davidson (1966) used data from previous studies in Toronto and London to calibrate a hyperbolic travel time function, in addition to the data he

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collected on one arterial road in Brisbane. Menon et al. (1974) calibrated Davidson's function against data they collected on two arterial roads in Sydney. Taylor (1977), also applied Davidson's function to two arterial roads sections with centre tram lines in Melborne. Overgaard (1967) calibrated an exponential function using the data collected in Toronto by Irwin and Von Cube in 1961.

Spiess (1990) developed the conical volume delay function formulation. This mathematical function provides a viable alternative formulation to the classical BPR functions. These BPR (Bureau of Public Roads) functions are the most widely used volume delay functions. The simplicity of the BPR formulation is one of the main reasons for its widespread use. The great advantage of the conical function alternative to the BPR on assignment procedures is a faster convergence in the user equilibrium assignment. The assigned link volumes and average travel times obtained with the conical function show almost no appreciable change against the BPR results. Appendix A is a summary of the findings of Spiess and the arguments used to propose the conical functions as a feasible alternative to the BPR formulation.

Evidence of faster convergence in equilibrium assignment with the conical function has been found in different studies. Through the Data Management Group of the Joint Program in Transportation at the University of Toronto, Cheah, Dalton and Hariri (1992) conducted a study called "Alternative Approaches to Volume Delay Function Formulation". The study examined the performance of several alternative formulations of volume delay
functions in relation to the BPR formula, in simulating auto volumes on the Greater Toronto Area road network. The study shows that the BPR formula is not well behaved when the network is congested (i.e. $v / c$ ratios $>1$ ). These functions become inadequate in terms of computational efficiency when long term (>20 years) forecast conditions are simulated and where the road networks become saturated. The functions analyzed were the conical, tangent, asymptotic, half tangent and horizontal functions. These five alternative formulations were individually tested using the equilibrium assignment procedure of EMME/2 under two different scenarios. These scenarios are the base year (1986) flows and a simulated over capacity condition where $150 \%$ of the base year flows are assigned to the same base year road network. The performance of each alternative volume delay function is measured by how well it can reproduce the assigned link volumes and average trip time results of the BPR function, as well as the speed of convergence (i.e. the number of iterations required to reach convergence). The results presented indicated that both the conical and tangent functions were the best alternative volume delay functions in terms of assigned link volumes and average trip time as well as speed of convergence.

Spiess (1984) shows a transportation study for the City of Basel, Switzerland, where the conical volume delay functions have been used successfully in practice. A dramatic improvement in the speed of convergence of the equilibrium assignment was observed when switching from the previously used BPR functions to the corresponding conical functions, with practically no significant changes in the network flows. The study was carried out using the EMME/2 transportation planning system.

## PARAMETER INVESTIGATION

The objective of this parameter investigation was to find particular road parameters that describe variation in observed travel times. The effects of flow and density parameters on the empirical travel time were also investigated with the analysis of empirical volumetravel time and density-travel time relationships under different road conditions. The size of the database and the variety of conditions under which the observations were taken allowed this project to investigate different explicit road parameters (different road characteristics). The database has also provided a well distributed range of flows that were useful for the analysis of empirical volume-travel time relationships. However, the information has a lack of observations in high density conditions and the range of flows studied was restricted to conditions below critical density. The critical density being a particular value of density used to differentiate between two density ranges, the under critical density conditions were density has a moderate influence on average travel time and the above critical density conditions were density has a significant influence on the average travel time.

A final division of the database was made with selected road parameters that have significant influence on the empirical travel times (presence of streetcars, speed limit, number of signalized intersections and headway between the buses). Categories were selected based on these parameters and were used to form groups with particular road conditions. Each of these groups presents specific travel time characteristics that corroborate the influence of the selected road parameters on the travel times. The
observations contained on these groups where used for the analysis of volume-travel time and density-travel time relationships

The available database was designed to represent the average behavior of all links in a given group and to allow investigation of the most important parameters influencing travel time. To conduct this investigation, it is necessary to recognize two main groups of parameters, the parameters defining traffic conditions and the parameters defining the road conditions. Density and volume are the two parameters used to define the traffic conditions and were investigated in this analysis. The objective was to observe how the fluctuations in volume and density influence travel times. It has been assumed that the volume-travel time and the density-travel time relationships have particular characteristics under specific road conditions. Therefore, it first was necessary to define a set of particular road conditions that allowed the analysis of the density-travel time and volume-travel time relationships. To define these particular road conditions, it was necessary to identify the road parameters that produce the most significant influence on travel time and then hold as many of these parameters constant as the empirical database allowed. The assumption was that if road parameters with significant influence on the travel time are held constant, then any relationships for different traffic conditions and travel time will be apparent. However, as more road parameters are held constant in the analysis, more groups are created and if more groups are created less information is available in each group. It was important to maintain a certain number of observations in each group formed to permit meaningful analysis on each parameter.

To identify a particular road parameter as significant, some categories based on values of the parameter must be investigated. These categories will be used to create different groups, which must contain a sufficient number of observations to show specific and well distributed volume-travel time and density-travel time relationships. If these relationships present specific travel time characteristics with reasonable relationships between observed travel times and the different road conditions specified, then we are able to corroborate a significant influence of the selected parameter on the travel time. As mentioned before the limited number of observations in the database restricts the number of parameters that could be held constant. Therefore, we need to arrange the order of importance of these road parameters to start dividing the database.

Using experience from previous studies and evidence from the empirical information, the road parameters of speed limit and presence of transit vehicles were selected as the most important road parameters and were the first parameters to be held constant. The values of these two parameters were used to form different categories, which were used to split the observations into six different groups. First, the presence of transit vehicles split the information to form two main groups representing road with streetcars and roads without streetcars. Then a second division was defined on the basis of speed limit of the road section where the observations were collected. On the group with no streetcars, five different speed limit groups were formed. The streetcar group has a reduced number of observations and just one speed limit was formed. The empirical volume-travel time and density-travel time plots resulting from this first division do not show clear relationships. However, the characteristic used to recognize the influence of the road parameters on
travel time was the average travel time of the observations on each of the groups which shows a reasonable distribution among these groups (Table 2.1).

These two road parameters were held constant and the influence of other road parameters on the empirical travel times was investigated. As previously mentioned, the limited number of observations in the database restricts the number of road parameters that could be held constant and the different groups that could be formed. The $70 \mathrm{~km} / \mathrm{hr}$ and the 80 $\mathrm{km} / \mathrm{hr}$ groups with no streetcars have a small number of observations that restricts further analysis. The investigation of the influence of different road parameters on the empirical travel times was carried out for the 40,50 and $60 \mathrm{~km} / \mathrm{hr}$ groups with no streetcars as well as on the $50 \mathrm{~km} / \mathrm{hr}$ groups with streetcars. The road parameters investigated were the following; number of signalized intersections per kilometer ( $\mathrm{s}+\mathrm{s} / \mathrm{km}$ ), lane width ( $\ln$ width), minimum lanes (minlanes), number of signalized intersections with possible through lane blockage due to left turning vehicle (LTB/km), development type (dev-type) and bus headways (hdw). The number of combinations that could be carried out with these different parameters and their different values to form different categories are enormous. Several combinations were investigated and were sufficient to recognize that the greatest influence on the empirical travel times was the number of signalized intersections per kilometer in addition to headway between buses.

The selection of these two road parameters required a definition of the appropriate values for these parameters that allow the creation of different categories that will create groups with sufficient information. The number of observations on each of the proposed groups
must be large enough to provide a range of volume and density measures in the volumetravel time and density travel time relationships. Fortunately, appropriate defining values for these parameters were found and the groups formed show reasonable and specific volume-travel time and density-travel time relationships.

The number of signalized intersections per kilometer parameter is well distributed among the observations and was used to create further divisions on the database. The values of 1.5 and 3 signals per kilometer were use to split the observations into three different categories of signal frequency. The $50 \mathrm{~km} / \mathrm{hr}$ streetcar group has insufficient data to form three signal frequency categories and just two categories were formed using the value of 3 signals per kilometer. Unfortunately, this split of signal frequency reduces the number of observations in the groups and just the $50 \mathrm{~km} / \mathrm{hr}$ and $60 \mathrm{~km} / \mathrm{hr}$ groups were left to study the influence of the headway between the buses. A significant influence of the bus headway parameter on travel time was found. To observe this influence clearly, the value of eight minutes was selected as a boundary to form two groups which are thought to represent high and low headways.

Finally the empirical volume-travel time and density-travel time relationships on each of the final groupings, present particular travel time characteristics due to the different road conditions represented. The specific travel time characteristics used to differentiate among the empirical volume-travel time relationships are the average travel time of observations in each group and the dispersion of the travel time data points from this average. The specific ranges of travel time dispersion observed appear to be constant over the different flow values in each of the different groups. However, the values for the dispersion of the
travel time form the average travel times in each of the groups have reasonable fluctuation with the different road conditions represented. Details of the final division and the trends found on each group are shown in the following section.

## Final Road Parameters Selected \& Details for the Division

The first parameter used to split the database was the presence of transit vehicles. This parameter split the database to form two main groups, streetcars with 194 observations and the bus+none group with 3278. Initially, three descriptive values of the presence of transit vehicle parameter were applied to the database; streetcars, bus, and no transit vehicle. The no transit vehicle group has a small number of observations (269 data points) which would restrict further detailed analysis. In addition, the empirical volume-travel time plots on the bus and no transit vehicles groups demonstrate very similar relationships. In order to obtain a group with a larger number of observations, these two groups were merged and the bus+none transit vehicle group was created. Figures 2.1 and 2.2 show the empirical volume travel time relationships on the transit vehicle groups formed.

The second road parameter used was the speed limit of the road sections. From all the observations in the database, nine different speed limits were found. Considering a tolerance of five kilometers per hour, the information was merged to form five speed limit groups that concentrate the number of observations. This action provides better conditions
for further analysis of other road parameters. The results obtained are shown in Table 2.1 and the volume-travel time relationships are shown in Figures 2.3 to 2.7.

| transit vehicle | speed limit | group name | data points | avg. travel time <br> (min $/ \mathrm{km})$ |
| :---: | :---: | :---: | :---: | :---: |
| bus+none | $40-45 \mathrm{~km} / \mathrm{hr}$ | $40 \mathrm{~km} / \mathrm{hr}$ | 342 | 2.03 |
|  | $50-55 \mathrm{~km} / \mathrm{hr}$ | $50 \mathrm{~km} / \mathrm{hr}$ | 1194 | 1.83 |
|  | $60-65 \mathrm{~km} / \mathrm{hr}$ | $60 \mathrm{~km} / \mathrm{hr}$ | 1324 | 1.54 |
|  | $70-75 \mathrm{~km} / \mathrm{hr}$ | $70 \mathrm{~km} / \mathrm{hr}$ | 234 | 1.43 |
|  | $80 \mathrm{~km} / \mathrm{hr}$ | $80 \mathrm{~km} / \mathrm{hr}$ | 184 | 1.08 |
| streetcar | $50 \mathrm{~km} / \mathrm{hr}$ | $50 \mathrm{~km} / \mathrm{hr}$ | 168 | 3.03 |

Table 2.1

The number of signalized intersections per kilometer is the third road parameter used to split the database. Several attempts were made to find appropriate values for this parameter to split the data. Finally, the values of 1.5 and 3.0 intersections per kilometer were used to form different groups. The volume-travel time relationships are shown in Figures 2.8 to 2.18 (pg. 26-36) and the division of the data is shown in Table 2.2

| transit vehicle | $\begin{gathered} \mathrm{spd} \\ (\mathrm{~km} / \mathrm{hr}) \end{gathered}$ | signals per km | data points | avg. travel time $(\mathrm{min} / \mathrm{km})$ |
| :---: | :---: | :---: | :---: | :---: |
| bus+none | 40 | s+s/km<1.5 | 124 | 1.81 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | 172 | 1.97 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | 46 | 2.84 |
|  | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | 350 | 1.44 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | 570 | 1.72 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | 274 | 2.47 |
|  | 60 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | 452 | 1.38 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | 824 | 1.57 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | 48 | 2.27 |
|  | 70 | no division | 234 | 1.43 |
|  | 80 | no division | 184 | 1.08 |
| streetcars | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | 80 | 2.25 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | 88 | 3.73 |

Table 2.2

Despite the small number of observations in the $70 \mathrm{~km} / \mathrm{hr}$ category, a signal frequency division was tried, but the travel time characteristics did not show sufficient difference between this and adjacent groups to support such a division. The roads with streetcars had a limited number of observations and the data points with signals per kilometer less than 1.5 were insufficient to form a group. Therefore, the information has been split into two categories and the value of 3 intersections per kilometer was used to create two groups. The definition of the $s+s / k m$ parameter in the database is the number of controlled intersections per kilometer. This is the number of signalized intersections per kilometer plus the stop signs encountered. The $80,70,60,50 \mathrm{~km} / \mathrm{hr}$ and streetcar groups have almost no stop signs, just the $40 \mathrm{~km} / \mathrm{hr}$ have 27 observations with stop signs, 102 with both stop signs and signals, and the remaining 213 observations are only signals. The empirical volume travel time relationships of these three $40 \mathrm{~km} / \mathrm{hr}$ groups were analyzed and no significant differences were found. Therefore, in order to increase the number of observations in the group, all observations were considered.

The fourth and last parameter to divide the database is the headway between the buses (measured in minutes). This road parameter was not included in the data collection and it was obtained from the transit schedules for the respective times and areas. Unfortunately, only the $50 \mathrm{~km} / \mathrm{hr}$ and the $60 \mathrm{~km} / \mathrm{hr}$ groups had sufficient data to afford the division. Analysis of the observations yielded the value of 8 minutes as a feasible limit to split the data points. The $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ for $60 \mathrm{~km} / \mathrm{hr}$ group has just 48 data points and could not support a headway division. The results for the division are shown in Table 2.3 and the volume delay plots in Figures 2.19 to 2.28 (pg. 37-46)

| transit vehicle | $\begin{gathered} \mathrm{spd}_{(\mathrm{km} / \mathrm{hr})} \end{gathered}$ | signals per km | hdw (min) | data points | avg. travel time(min/km) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| bus+none |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ |  | 124 | 1.81 |
|  | 40 | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 172 | 1.97 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ |  | 46 | 2.84 |
|  | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 272 | 1.42 |
|  |  |  | hdw<8 | 78 | 1.51 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | hdw>8 | 378 | 1.63 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | hdw<8 | 192 | 1.98 |
|  |  |  | hdw>8 | 166 | 2.43 |
|  |  |  | hdw<8 | 108 | 2.61 |
|  | 60 | $\begin{gathered} \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5 \\ 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3 \\ \mathrm{~s}+\mathrm{s} / \mathrm{km}>3 \end{gathered}$ | hdw>8 | 354 | 1.36 |
|  |  |  | hdw<8 | 98 | 1.43 |
|  |  |  | hdw>8 | 382 | 1.54 |
|  |  |  | hdw<8 | 442 | 1.62 |
|  |  |  | no division | 48 | 2.27 |
|  | 70 | no division | no division | 234 | 1.43 |
|  | 80 | no division | no division | 184 | 1.08 |
| streetcars | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 80 | 2.25 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | no division | 88 | 3.73 |

Table 2.3

The empirical volume-travel time relationships show that travel times remain constant over a wide range of flows for all the different groups of observations. The average travel time and the dispersion of data points from the average travel time have reasonable fluctuation on each particular group according to the different road conditions represented. The travel time characteristics of the different groupings are analyzed in the following section. The objective is to represent the observed behavior with a mathematical formulation.

Figure 2.1
T vs Q (bus+none)
3278 points : $1.66 \mathrm{~min} / \mathrm{km}$


Figure 2.2
T vs Q(streetcar)
194 points : $2.98 \mathrm{~min} / \mathrm{km}$


Figure 2.3
T vs Q (spd $40-45 \mathrm{~km} / \mathrm{hr}$ )
342 points : $2.03 \mathrm{~min} / \mathrm{km}$


Figure 2.4
T vs Q (spd $50-55 \mathrm{~km} / \mathrm{hr}$ )
1194 points : $1.83 \mathrm{~min} / \mathrm{km}$


Figure 2.5
T vs Q (spd $60-65 \mathrm{~km} / \mathrm{hr}$ ) 1324 points : $1.54 \mathrm{~min} / \mathrm{km}$


Figure 2.6
T vs Q (spd $70-75 \mathrm{~km} / \mathrm{hr}$ ) 234 points : $1.43 \mathrm{~min} / \mathrm{km}$


Figure 2.7
T vs Q (spd $80 \mathrm{~km} / \mathrm{hr}$ )
184 points : $1.08 \mathrm{~min} / \mathrm{km}$


Figure2.8
$T$ vs $Q($ spd $40 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5)$ 124 points : $1.81 \mathrm{~min} / \mathrm{km}$


Figure 2.9
T vs $Q($ spd $40 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$ )
172 points : $1.97 \mathrm{~min} / \mathrm{km}$


Figure 2.10
T vs Q(spd $40 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )
46 points : $2.84 \mathrm{~min} / \mathrm{km}$


Figure 2.11
T vs Q (spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ )
350 points: $1.44 \mathrm{~min} / \mathrm{km}$


Figure 2.12
T vs Q (spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ )
570 points : $1.72 \mathrm{~min} / \mathrm{km}$


Figure 2.13
T vs Q (spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )
274 points : $2.47 \mathrm{~min} / \mathrm{km}$


Figure 2.14
T vs Q (spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ ) 452 points : $1.38 \mathrm{~min} / \mathrm{km}$


Figure 2.15
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ )
824 points : $1.58 \mathrm{~min} / \mathrm{km}$


Figure 2.16
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )
48 points : $2.27 \mathrm{~min} / \mathrm{km}$


Figure 2.17
T vs $Q$ (st, spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<3$ ) 80 points : $2.25 \mathrm{~min} / \mathrm{km}$


Figure 2.18
T vs Q (st, spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )
88 points : $3.73 \mathrm{~min} / \mathrm{km}$


Figure 2.19
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $>8$ )
272 points : $1.42 \mathrm{~min} / \mathrm{km}$


Figure 2.20
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $\mathrm{h}<8$ )
78 points : $1.51 \mathrm{~min} / \mathrm{km}$


Figure 2.21
T vs $Q$ (spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$, hdw $>8$ )
378 points : $1.63 \mathrm{~min} / \mathrm{km}$


Figure2.22
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0, \mathrm{hdw}<8)$
192 points : $1.98 \mathrm{~min} / \mathrm{km}$


Figure 2.23
T vs $Q$ (spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, hdw $>8$ )
166 points : $2.43 \mathrm{~min} / \mathrm{km}$


Figure 2.24
$T$ vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, hdw $<8$ )
108 points : $2.61 \mathrm{~min} / \mathrm{km}$


Figure 2.25
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $>8$ )
354 points : $1.36 \mathrm{~min} / \mathrm{km}$


Figure 2.26
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw<8)
98 points : $1.43 \mathrm{~min} / \mathrm{km}$


Figure 2.27
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$, hdw h 8 )
382 points : $1.54 \mathrm{~min} / \mathrm{km}$


T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$, hdw $<8$ )
442 points : $1.62 \mathrm{~min} / \mathrm{km}$


## DATA ANALYSIS \& CALIBRATION

The main objective of the data analysis was to calibrate the parameters of the conical volume delay function to represent the observed volume-travel time relationships. Conical curves were fit to every group described in the previous section. The form of the conical curve in the range of flows less than the critical flow is a reasonable representation of the empirical volume-travel time relationships in each of the groups. Representations of volume travel time relationships over the critical flow value are more problematic and are discussed later. The calibrated parameters appear to represent the particular road conditions of each group.

As described in the previous section, eighteen different groups were formed, each of these groups have particular road conditions and present specific travel time characteristics under different traffic conditions. Unfortunately, the majority of the observations in the database represent traffic conditions less than critical density. Most of the densities are below forty vehicles per kilometer per lane. The database contains a very small number of observations at high travel times and high densities. It is clear that extremely high densities dramatically increase the travel time on a road section. This project assumed that at a certain value of density (critical density), the volume travel time relationship behavior will change. However, the range of densities in the database was insufficient to determine an accurate value for these critical densities.

In general, for the under critical density conditions observed, it could be said that the travel time is independent of flow. In all the groups formed, the under critical density ranges of
flow show almost constant travel times. It would appear that the empirical volume-travel time relationships could be modeled with a linear function that is either horizontal or has a very small positive slope. However, the average travel time of the observations and the variation in travel time within each of the groups, present consistent variation that is consistent with the specific road conditions.

The presence of streetcars on a road section creates a significant increase in the average travel time. The speed limit parameter shows that greater restrictions of speed on a road section increase the average travel time. The groups with a higher number of signalized intersections per kilometer show a higher average travel time and a greater dispersion of data points from the average travel times. Also, the 50 and 60 kilometer per hour bus+none groups in the different signal frequency groups show that the lower headway groups produce higher average travel times with an increase in the variation in travel time when compared with the high headway groups. Unfortunately, in the 50 kilometer per hour group with the lower signal frequency, the group with lower headways does not show the increased variation of travel time due to the limited number of observations. It is also important to mention that the particular group with streetcars and high signal frequency demonstrate a travel time relationship that is distinct from the other groups.

The conical volume delay function was selected from a set of different volume-travel time functions as a feasible formulation. The under critical flow form of the conical curve is very similar to the observed volume-travel time relationships and could be used to simulate the proposed range of road conditions. The conical formulation has been developed as a
viable alternative to the widely accepted BPR function. This wide acceptance of the BPR function is due to the simplicity of the formulation and the ability of the function to provide satisfactory results in the assignment procedures. The particular mathematical properties of the conical function according to Spiess, overcome the "inherent drawbacks" of the BPR formulation (see Appendix A). Evidence from different studies show successful applications of conical functions which have provided faster convergence in equilibrium assignments and almost no change on network flows previously estimated with the BPR formulation.

The form of a conical curve can be roughly described on two main sections, below and above the critical flow. The critical flow is defined as the maximum value of flow that could pass on a particular road section without the traffic density being so great as to cause unreasonable travel time. The form of the conical curve section below this critical flow has a portion that is roughly similar to a linear function with a smooth slope. This under critical flow shape could be adapted to represent the empirical volume-travel time relationships by the calibration of the conical function parameters. It is also important to mention that the section above the critical flow value is an unrealistic representation used to satisfy the assignment requirements. The conical volume delay function has three parameters that define its form, alpha ( $\alpha$ ), critical flow (c), and free flow travel time (t0). Appropriate values for these parameters need to be found in order to represent the empirical relationships observed.

As described above, it would appear that the best representation of the volume travel time relationships for under critical density conditions is a linear function. Therefore, a least
square fit with a linear function on observations in the different groups was carried out. The resulting linear volume-travel time functions tend to be horizontal or have a very small positive slope. These estimated straight lines are very close to a line with no slope running over the average travel time (Figures 3.3 to 3.20 ).

The $70 \mathrm{~km} / \mathrm{hr}$ group has a significant number of high travel times with high density observations, which clearly causes the linear regression line to divert considerably from the horizontal (Figure 3.17). These observations correspond to an "upper critical density" volume-travel time relationship, completely different from the "lower critical density" form. Therefore, these observations will not be considered in further analysis. A number of high density-high travel time observations were also removed from further analysis in the high headway group of the low signal frequency group of the $60 \mathrm{~km} / \mathrm{hr}$. Other groups also contain these high travel time-high density observations but the number of data points was small and the effect of these observations on the analysis was considered not to be significant.

As described above, the under critical flow section of the conical curve has a portion roughly similar to a straight line that could be adapted to imitate the linear behavior of the observed volume-travel times. However, if small values of alpha are used (i.e. 2 or 4 ) the form of the conical curve divert from this linear behavior (see figure 3.2 A). On the contrary, if high values of alpha are use the form of the conical curve became more similar to a straight line. In order to represent the volume travel time behavior in the under critical flow section of the conical curves, large alpha values (i.e. 10 or 12) should be used but
these alpha values also influence the upper critical flow form of the curve (see Figure 3.2 B).

In a first approach to imitate the empirical volume travel time relationships with the conical function, the values for the alpha and the critical flow parameters were determined by a least squares fit. First the average travel time values in each group were used for the t 0 conical function parameters in order to imitate the empirical relationships with the conical functions. Then a least square fit was used to calibrate the alpha and critical flow values. The results obtained were unreasonable and the approach was rejected. Extremely high alpha values were found (i.e. 40), clearly the regression procedure with the conical curve was trying to imitate the straight line behavior well beyond the range of observed values. A value for the alpha parameter was required that would provide a relatively linear initial portion of the curve for under critical flow conditions and before increasing rapidly on the over critical flow section of the conical curve. The alpha value of 6 was successful on fulfilling the former requirements and has been used for all the functions.

A second attempt at least square fitting was carried out with the above constraint on alpha. The least squares fit was carried out for different groups and the results yield senseless critical flow values. If these high critical flows were used on the conical function, the under critical flow section most similar to a linear behavior is extended to the majority of the observations and the least square procedure obtains a better fit. Clearly, again, the least square fit results tend to imitate the straight line behavior well beyond the range of observed values. The extremely high critical flow values obtained make this second approach unsatisfactory.

A third approach proved to be successful. It was divided into three main parts. In the first part, the average travel time of the observations on the groups was used again for the value of the t 0 conical function parameters in order to imitate the initial straight line behavior. This time the fixed value of the alpha parameter (equal to 6) set appropriate conditions to visually select feasible values for the critical flow parameters. As mentioned before, the critical flow value divides the conical curves into two sections with different volume-travel time behavior. Following the assumption of the critical density, these two sections of the conical curve have been considered as the under and over critical density sections. The limited number of high travel time-high density observations in the groups do not provide enough evidence to clearly observe the change of behavior from below to above the critical flow in the empirical volume-travel time relationship. However, these empirical volumetravel time observations are well distributed among the under critical density section and an approach to the critical flow value can be to look at the upper limit of observed flows. Within a range of a $100 \mathrm{veh} / \mathrm{hr} / \mathrm{lane}$, three feasible consecutive values were selected and the middle option chosen as a feasible critical flow value. The reasonableness of the result among the different groups were use to corroborate the findings (Figure 3.21). The results show an increase in the critical flow values when the speed limit is increased. It seems that the values for critical flow decrease in some cases when the number of signalized intersections increase. Also, in some cases the higher headway groups have greater critical flow values.

In the second part of the calibration process, least square procedures were undertaken to obtain the t 0 conical function parameters. The previous decision of setting these t 0
parameters to the average travel time of the observations of each particular group does not provide an optimum fit. An improvement in these fits of the conical curves to the empirical observations on each of the groups was needed. The fixed values of the critical flows allowed least square procedures to obtain new t0 parameters that will provide better fits. The results for the t 0 parameters on these least square procedures yielded smaller values than the previously used average travel times. These smaller values cause the conical curves to move down vertically, changing the earlier conditions observed for the selection of critical flow values. Therefore, some of the critical flows previously adjusted were no longer valid and needed to be reevaluated.

On the third part of this calibration process, a new estimation for the critical flow values was carried out using the same criteria described previously. Four of the groups did not require a change to their critical flow values but for the remainder of the groups, new values were estimated. By doing so, the shapes of the curves change again and, therefore, the previous least square fit for the $t 0$ conical parameters needed to be repeated. For the second time, least square procedures were carried out to obtain the t0 parameters. This time the new free flow travel time values represent a very small variation from the previous calculations. These small variations did not cause significant changes to the curves. Nevertheless, a final inspection of the critical flow values was carried out and showed no need for correction to the values. Finally, the values of the calibrated parameters were estimated.

## Details of the Calibration \& Final Results

As mention previously, the process used to calibrate the free flow travel time and the critical flow parameters of the conical volume delay functions can be divided into three main parts. The details of the calculations on these three parts and the final results obtained are explained in the following.
I. The Free Flow Travel Time Conical Parameters takes the Average Travel Time Values The conical parameter of alpha was assigned with the value of six to represent an average behavior on the conical volume delay functions. The $t 0$ parameter values were assigned the average travel time of the observations in each group. This was done to approximate the initial linear behavior of the empirical observations. The $c$ parameters were calculated on each of the groups by looking at the plots of the volume-travel time observations with the conical curve. Three different consecutive feasible values were selected with an accuracy of a $100 \mathrm{veh} / \mathrm{km} / \mathrm{lane}(\mathrm{e} . \mathrm{g} .800,900,1000$ ) and the one in the middle of them was chosen to represent the critical flow for the specific group. Figures 3.3 to 3.20 (pg. 60-77) show the results of the analysis. The plots show the data points included in the group, the least square regression line, the line of the average of the observations and the conical volume delay adjusted function. The selected critical flow value is shown in the legend. Figure 3.21 (pg. 78) provides a summary of the results for all the groups.

The initial results for the $70 \mathrm{~km} / \mathrm{hr}$ group indicated an average travel time of $1.43 \mathrm{~min} / \mathrm{km}$ which is essentially the same as the $60 \mathrm{~km} / \mathrm{hr}$ group with low signals per kilometer and low headways. This is not a reasonable result. As mention previously, twelve data points from
the $70 \mathrm{~km} / \mathrm{hr}$ group are clearly out of the constant travel time range. These observations have high travel time at high density values and are part of a different volume travel time behavior under high density conditions. Therefore, these twelve data points have been excluded from the calibration procedure. The average travel time on the group without these points yields a more reasonable result of $1.21 \mathrm{~min} / \mathrm{km}$. The same procedure was done in the $60 \mathrm{~km} / \mathrm{hr}$ group with low signals per kilometer and high headways where six observations with high travel times at high density conditions were excluded. Other groups also have high travel times under high density conditions but the reduced number of these observations (usually no more than two) do not produce significant effects on the calibration.

## II. Free Flow Travel Times Determined by Least Square Procedures

The use of the average travel times of observations in a group for the free flow travel time parameters was done to represent the linear behavior of the data points and estimate the critical flow values on a conical curve. This does not provide an optimal fit of the conical curve to the observations and therefore is not an optimal solution for the $t 0$ parameter values. The calculation of feasible values for the critical flows permitted least square analysis that would let the observations determine the optimum values for the $t 0$ parameters. Least square procedures were carried out for all the groups to calculate the $t 0$ parameters, the results are shown in Figures 3.22 to 3.39 (pg. 79-96) and Figure 3.40 (pg. 97) show all the values for the different groups.

The results show a significant decrease from the previously used $t 0$ values. This caused the conical curve to move down vertically and therefore the conditions observed for the
estimation of the critical flow parameters in the previous section have been changed. A new inspection is needed to estimate appropriate values for these critical flow values.

## III. Adjustment of the Critical Flow Values and New Least Square Procedures

A new selection of the critical flow values was necessary for some of the groups. In fourteen out of eighteen groups, the critical flow value was reduced by $100 \mathrm{veh} / \mathrm{hr} / \mathrm{lane}$ in an attempt to compensate for the change in the $t 0$ value. This change in the critical flow values reshaped the form of the curves again and therefore the previous least square analysis no longer provides an optimum fit. A second least square analysis was carried out to obtain new $t 0$ values. This time the new t0 parameters do not show significant changes against the previously used values. The plots of the observations were tested again to confirm that no further changes were needed in the critical flow values. Figures 3.41 to 3.54 (pg. 98-111) show the results on each of the groups and Figure 3.55 (pg.112) summarizes the final calibrated parameters.

## Final Results

The results from the calibration process (fig. 3.55) were acceptable and, in general, the reasonableness of the values among the different groups provides confidence in the process. However, one inconsistency was found in the results, the $t 0$ value for the 60 $\mathrm{km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, $\mathrm{hdw}<8$ has a relatively high t 0 value of $1.3 \mathrm{~min} / \mathrm{km}$. It has been assumed that this inconsistency is due to the reduced number of observations in the group combined with an insufficient range of flows. This particular group has 98 data points in comparison to more than 350 on the other groups of the $60 \mathrm{~km} / \mathrm{hr}$, also the distribution of
flow values in the group ranges from 200 to 800 where the other groups have a greater range of flows form 200 to 1200 . Following the evidence for the $50 \mathrm{~km} / \mathrm{hr}$ group, it seems that the effect of the headways on changing the form of the conical curve is not evident on the small signal frequency groups. Therefore, it is recommended to use a value of $1.1 \mathrm{~min} / \mathrm{km}$ instead of the $1.3 \mathrm{~min} / \mathrm{km}$ estimated in the analysis. The final values proposed for the parameter of the different conical functions are contained in the following Table (3.1).

| transit <br> vehicle | $\begin{array}{\|c\|} \hline \mathrm{spd} \\ (\mathrm{~km} / \mathrm{hr}) \end{array}$ | signals per km | hdw (min) | c (veh/hr/lane) | to ( $\mathrm{min} / \mathrm{km}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| bus+none |  | s+s/km<1.5 |  | 900 | 1.5 |
|  | 40 | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 700 | 1.7 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ |  | 700 | 2.4 |
|  | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 1000 | 1.3 |
|  |  |  | hdw<8 | 1000 | 1.3 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | hdw>8 | 1000 | 1.4 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | hdw<8 | 900 | 1.4 |
|  |  |  | hdw>8 | 900 | 2 |
|  |  |  | hdw<8 | 800 | 2 |
|  | 60 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 1100 | 1.1 |
|  |  |  | hdw $<8$ | 1100 | 1.1 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | hdw>8 | 1100 | 1.2 |
|  |  |  | hdw<8 | 1000 | 1.2 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | no division | 1000 | 1.7 |
|  | 70 | no division | no division | 1200 | 1 |
|  | 80 | no division | no division | 1300 | 0.9 |
| streetcars | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 900 | 1.8 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | no division | 800 | 3.3 |

Table 3.1

Appendix B shows the plots of the different volume travel time functions proposed.

Figure 3.1 (A, B) BPR Functions



Figure 3.2 (A, B)
Conical Functions


B

Figure 3.3
$T$ vs $Q($ spd $40 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5)$ 124 points : $1.81 \mathrm{~min} / \mathrm{km}$


- observations $\quad$ conic vdf $(\mathrm{c}=900) \triangle$ average —linear least square fit

Figure 3.4
T vs $Q($ spd $40 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0)$
172 points : $1.97 \mathrm{~min} / \mathrm{km}$


Figure 3.5
T vs Q(spd $40 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ ) 46 points : $2.84 \mathrm{~min} / \mathrm{km}$


Figure 3.6
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $>8$ ) 272 points : $1.42 \mathrm{~min} / \mathrm{km}$


- observations
- conic vdf (c=1100) average $\qquad$

Figure 3.7
$T$ vs $Q(s p d 50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, $\mathrm{hdw}<8$ )
78 points : $1.51 \mathrm{~min} / \mathrm{km}$


- observations $\quad$ conical vdf $(\mathrm{c}=1100) \quad$ average ——linear least square fit

Figure 3.8
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0, \mathrm{hdw}>8$ ) 378 points : $1.63 \mathrm{~min} / \mathrm{km}$


Figure 3.9
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0, \mathrm{hdw}<8$ )
192 points : $1.98 \mathrm{~min} / \mathrm{km}$

observations

- conic vdf ( c=1000)
average ——linear least square fit

Figure 3.10
$T$ vs $Q$ (spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, hdw>8) 166 points : $2.43 \mathrm{~min} / \mathrm{km}$


- observations - conic vdf ( $\mathrm{c}=1000$ ) $\triangle$ average ——linear least square fit

Figure 3.11
T vs Q (spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, hdw<8)
108 points : $2.61 \mathrm{~min} / \mathrm{km}$


- observations
conical vdf (c=900)
average -linear least square fit

Figure 3.12
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $>8$ )
354 points : $1.32 \mathrm{~min} / \mathrm{km}$


- observations $\quad$ conic vdf $(\mathrm{c}=1200) \triangle$ average ——linear least square fit

Figure 3.13
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw<8)
98 points : $1.43 \mathrm{~min} / \mathrm{km}$


- observations $\quad$ conic vdf $(\mathrm{c}=1100) \triangle$ average ——linear least square fit

Figure 3.14
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$, hdw>8)
382 points : $1.54 \mathrm{~min} / \mathrm{km}$


Figure 3.15
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$, hdw $<8$ )
442 points : $1.62 \mathrm{~min} / \mathrm{km}$


- observations ■ conic vdf $(\mathrm{c}=1100) \triangle$ average ——linear least square fit

Figure 3.16
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )
48 points : $2.27 \mathrm{~min} / \mathrm{km}$


- observations ■ conic vdf (c=1100) a average -linear least square fit

Figure 3.17
T vs Q (spd $70-75 \mathrm{~km} / \mathrm{hr}$ ) 243 points : $1.21 \mathrm{~min} / \mathrm{km}$


Figure 3.18
T vs Q (spd $80 \mathrm{~km} / \mathrm{hr}$ )
184 points : $1.08 \mathrm{~min} / \mathrm{km}$


- observations ■ conic vdf ( $\mathrm{c}=1400$ ) $\triangle$ average ——linear least square fit

Figure 3.19
T vs Q (st, spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<3$ ) 80 points : $2.25 \mathrm{~min} / \mathrm{km}$


- observations conic vdf (c=1000) average —linear least square fit

Figure 3.20
T vs Q (st, spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ ) 88 points : $3.73 \mathrm{~min} / \mathrm{km}$


Figure 3.21

| transit vehicle | spd (km/hr) | signals per km | hdw (min) | fc (veh/hr/lane) | to $=\mathrm{avg}(\mathrm{min} / \mathrm{km})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| bus+none |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ |  | 900 | 1.81 |
|  | 40 | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 800 | 1.97 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ |  | 700 | 2.84 |
|  | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 1100 | 1.42 |
|  |  |  | hdw<8 | 1100 | 1.51 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | hdw>8 | 1100 | 1.63 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | hdw<8 | 1000 | 1.98 |
|  |  |  | hdw>8 | 1000 | 2.43 |
|  |  |  | hdw<8 | 900 | 2.61 |
|  | 60 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 1200 | 1.32 *(1) |
|  |  | $\begin{gathered} 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3 \\ \mathrm{~s}+\mathrm{s} / \mathrm{km}>3 \end{gathered}$ | hdw<8 | 1100 | 1.43 |
|  |  |  | hdw>8 | 1200 | 1.54 |
|  |  |  | hdw<8 | 1100 | 1.62 |
|  |  |  | no division | 1100 | 2.27 |
|  | 70 | no division | no division | 1300 | 1.21 *(2) |
|  | 80 | no division | no division | 1400 | 1.08 |
| streetcars | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 1000 | 2.25 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | no division | 800 | 3.73 |

* (1) 4 "congested" data points were not considered for the average
* (2) 12 "congested" data points were not considered for the average

Figure 3.22
T vs $Q($ spd $40 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ )
alpha=6, $c=900, \mathrm{t} 0=1.5$


Figure 3.23
T vs $Q($ spd $40 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$ )
alpha $=6, \mathrm{c}=800, \mathrm{t} 0=1.8$


Figure 3.24
T vs $Q(s p d 40 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ ) alpha $=6, \mathrm{c}=700, \mathrm{t} 0=2.4$


Figure 3.25
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $>8$ )
alpha $=6, c=1100, t 0=1.3$


Figure 3.26
$T$ vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5, \mathrm{hdw}<8)$
alpha=6, $c=1100, \mathrm{t}=1.4$


Figure 3.27
$T$ vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0, \mathrm{hdw}>8$ )
alpha $=6, \mathrm{c}=1100, \mathrm{t} 0=1.5$


Figure 3.28
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0, \mathrm{hdw}<8$ )
alpha $=6, \mathrm{c}=1000, \mathrm{t}=1.6$


Figure 3.29
$T$ vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, hdw>8)
alpha $=6, \mathrm{c}=1000, \mathrm{t} 0=2.1$


Figure 3.30
$T$ vs $Q$ (spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, hdw<8)
alpha $=6, \mathrm{c}=900, \mathrm{t}=2.2$


Figure 3.31
T vs Q (spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $>8$ )
alpha $=6, \mathrm{c}=1200, \mathrm{t} 0=1.2$


Figure 3.32
$T$ vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $\mathrm{h}<8$ )
alpha $=6, \mathrm{c}=1100, \mathrm{t} 0=1.3$


Figure 3.33
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$, hdw h 8 ) alpha $=6, \mathrm{c}=1200, \mathrm{t} 0=1.3$


Figure 3.34
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$, hdw $<8$ )
alpha $=6, \mathrm{c}=1100, \mathrm{t} 0=1.3$


Figure 3.35
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )
alpha $=6, \mathrm{c}=1100, \mathrm{t} 0=1.9$


Figure 3.36
T vs Q (spd $70 \mathrm{~km} / \mathrm{hr}$ )
alpha $=6, \mathrm{c}=1300, \mathrm{t} 0=1.1$


Figure 3.37
T vs Q (spd $80 \mathrm{~km} / \mathrm{hr}$ )
alpha $=6, \mathrm{c}=1400, \mathrm{t} 0=1.0$


Figure 3.38
T vs Q (st, spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<3$ ) alpha $=6, c=1000, t 0=2.0$


Figure 3.39
T vs Q (st, spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )
alpha $=6, \mathrm{c}=800, \mathrm{t} 0=3.3$


Figure 3.40

| transit vehicle | spd (km/hr) | signals per km | hdw (min) | fc (veh/hr/lane) | to (min/km) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| bus+none |  | s+s/km<1.5 |  | 900 | 1.5 |
|  | 40 | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 800 | 1.8 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ |  | 700 | 2.4 |
|  | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 1100 | 1.3 |
|  |  |  | hdw<8 | 1100 | 1.4 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | hdw>8 | 1100 | 1.5 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | hdw<8 | 1000 | 1.6 |
|  |  |  | hdw>8 | 1000 | 2.1 |
|  |  |  | hdw<8 | 900 | 2.2 |
|  | 60 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 1200 | 1.2 |
|  |  |  | hdw<8 | 1100 | 1.3 |
|  |  | $\begin{gathered} 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3 \\ \mathrm{~s}+\mathrm{s} / \mathrm{km}>3 \end{gathered}$ | hdw>8 | 1200 | 1.3 |
|  |  |  | hdw<8 | 1100 | 1.3 |
|  |  |  | no division | 1100 | 1.9 |
|  | 70 | no division | no division | 1300 | 1.1 |
|  | 80 | no division | no division | 1400 | 1 |
| streetcars | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 1000 | 2 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | no division | 800 | 3.3 |

Figure 3.41
T vs $Q($ spd $40 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$ )
alpha $=6, \mathrm{c}=700, \mathrm{t} 0=1.7$


Figure 3.42
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $>8$ )
alpha $=6, c=1000, t 0=1.3$


Figure 3.43
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5, \mathrm{hdw}<8$ )
alpha $=6, \mathrm{c}=1000, \mathrm{t} 0=1.3$


Figure 3.44
$T$ vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$, $\mathrm{hdw}>8$ )
alpha $=6, \mathrm{c}=1000, \mathrm{t}=1.4$


Figure 3.45
T vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0, \mathrm{hdw}<8)$
alpha $=6, \mathrm{c}=900, \mathrm{t}=1.4$


Figure 3.46
$T$ vs $Q($ spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, $\mathrm{hdw}>8$ )
alpha=6, $c=900, t 0=2.0$


Figure 3.47
T vs $Q$ (spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, hdw<8)
alpha $=6, \mathrm{c}=800, \mathrm{t} 0=2.0$


Figure 3.48
T vs Q (spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $>8$ )
alpha $=6, \mathrm{c}=1100, \mathrm{t}=1.1$


Figure 3.49
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$, hdw $\mathrm{h}>8$ ) alpha $=6, \mathrm{c}=1100, \mathrm{t} 0=1.2$


Figure 3.50
T vs $\mathrm{Q}(\mathrm{spd} 60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$, hdw $\mathrm{h}<8$ )
alpha $=6, \mathrm{c}=1000, \mathrm{t}=1.2$


Figure 3.51
T vs $Q($ spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ ) alpha $=6, c=1000, t 0=1.7$


Figure 3.52
T vs Q (spd $70 \mathrm{~km} / \mathrm{hr}$ )
alpha $=6, \mathrm{c}=1200, \mathrm{t}=1.0$


Figure 3.53
T vs Q (spd $80 \mathrm{~km} / \mathrm{hr}$ )
alpha $=6, \mathrm{c}=1300, \mathrm{t} 0=0.9$


Figure 3.54
T vs Q (st, spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<3$ )
alpha $=6, \mathrm{c}=900, \mathrm{t} 0=1.8$


Figure 3.55
III

| transit vehicle | spd (km/hr) | signals per km | hdw (min) | fc (veh/hr/lane) | to ( $\mathrm{min} / \mathrm{km}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| bus+none | 40 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | no division | 900 | 1.5 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ |  | 700 | 1.7 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ |  | 700 | 2.4 |
|  | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 1000 | 1.3 |
|  |  |  | hdw<8 | 1000 | 1.3 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | hdw>8 | 1000 | 1.4 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | hdw $<8$ | 900 | 1.4 |
|  |  |  | hdw>8 | 900 | 2 |
|  |  |  | hdw<8 | 800 | 2 |
|  | 60 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ | hdw>8 | 1100 | 1.1 |
|  |  | $1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | hdw<8 | 1100 | 1.3 |
|  |  |  | hdw>8 | 1100 | 1.2 |
|  |  |  | hdw<8 | 1000 | 1.2 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | no division | 1000 | 1.7 |
|  | 70 | no division | no division | 1200 | 1 |
|  | 80 | no division | no division | 1300 | 0.9 |
| streetcars | 50 | $\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ | no division | 900 | 1.8 |
|  |  | $\mathrm{s}+\mathrm{s} / \mathrm{km}>3$ | no division | 800 | 3.3 |

## CONCLUSIONS

This project provides a consistent series of conical volume travel time functions capable of representing the empirical travel times for under critical density traffic conditions for a specific set of road characteristics. These road characteristics represent some of the most significant road parameters influencing travel time in the empirical study of signalized intersections on the GTA road network. There is evidence that the mathematical properties of these conical formulations could provide significant computational efficiencies on the user-equilibrium assignment procedures.

The analysis of the influence that specific road and traffic parameters produce on the empirical travel time, produces the following general conclusions. The particular road conditions determined the most significant road parameters, which are; the presence of transit vehicles, speed limits, number of signalized intersections per kilometer and headway between buses.

The selection of specific categories for these significant road parameters allowed the creation of specific groups where volume-travel time and density-travel time empirical relationships show particular travel time characteristics. For under critical density conditions, traffic flow does not produce a significant influence on empirical travel times. Traffic density in under critical density conditions produces moderate influence on the empirical travel times. However, this influence seems to increase according to the particular road conditions determined.

The analysis of empirical relationships is restricted to under critical density conditions. The great majority of observations in the database have densities below forty vehicles per kilometer per lane. The information in the database contains a very small number of observations with high travel times and high densities. However, it is assumed that the behavior of the empirical volume-travel time and density-travel time relationships for over critical density conditions differ considerably from the behavior of the relationships on under critical density conditions. Unfortunately the distribution of densities for the empirical observations was not sufficiently spread to make an accurate determination of critical density values and also limits the identification of specific relationships in over critical density conditions. Some reasonable assumptions were necessary to define critical density values.

The volume-travel time relationships observed in the database have sufficient information for under critical density conditions. The average travel times and the variation of these travel times were use as travel time characteristics, which are a reasonable representation of how different road conditions influence travel time.

The presence of streetcars creates a significant increase in the average travel time. The speed limit parameter shows that greater restrictions on speed increase the average travel time. The groups with higher number of signalized intersections per kilometer show higher average travel times and a greater variation in the travel times. Also the lower bus headway groups produce higher average travel times with increased travel time dispersion.

The successful adaptation of the conical curves to the empirical volume-travel time relationship on the different groups allows the calibration of the conical function parameters. These calibrated parameters are used to develop conical functions that represent the travel time relationships under various flow conditions for each group. The specific values for the calibrated parameters capture the influence of combinations of specific road parameters on the empirical travel times.

It is recommended to apply these series of volume-travel time functions to the electronic network of the GTA to analyze potential improvements to the assignment procedures.

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Appendix A,
Conical Travel Time Functions for the GTA
(spd40 km/hr)


Appendix A,
Conical Travel Time Functions for the GTA
(spd50 km/hr)


Appendix A,
Conical Travel Time Functions for the GTA
(spd $60 \mathrm{~km} / \mathrm{hr}$ )


Appendix A,
Conical Volume Travel Functions for the GTA
(spd $70 \& 80 \mathrm{~km} / \mathrm{hr}$ )


Appendix A,
Conical Travel Time Functions for the GTA
(streetcar, spd $50 \mathrm{~km} / \mathrm{hr}$ )


Appendix A,
Conical Travel Time Functions for the GTA
(all groups)


Appendix B
T vs K (spd $40 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$ )


Appendix B
T vs K (spd $40 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$ )


Appendix B
T vs K (spd $40 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )


Appendix B
T vs $\mathrm{K}(\mathrm{spd} 50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, $\mathrm{hdw}>8$ )


Appendix B
T vs K (spd $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw<8)


Appendix B
T vs $\mathrm{K}(\mathrm{spd} 50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$, hdw>8)


Appendix B
T vs K (spd $50 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$, hdw $<8$ )


Appendix B
T vs $\mathrm{K}(\mathrm{spd} 50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, $h d w>8$ )


Appendix B
T vs $\mathrm{K}(\mathrm{spd} 50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$, hdw<8)


Appendix B
T vs K (spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw>8)


Appendix B
T vs K ( spd $60 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<1.5$, hdw $<8$ )


Appendix B
T vs $\mathrm{K}(\mathrm{spd} 60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3.0$, $\mathrm{hdw}>8$ )


Appendix B
T vs K (spd $60 \mathrm{~km} / \mathrm{hr}, 1.5<\mathrm{s}+\mathrm{s} / \mathrm{km}<3$, hdw<8)


Appendix B
T vs K (spd 60km/hr, s+s/km>3)


Appendix B
T vs K (spd $70 \mathrm{~km} / \mathrm{hr}$ )


Appendix B
T vs K (spd $80 \mathrm{~km} / \mathrm{hr}$ )


Appendix B
T vs K (st, $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}<3$ )


Appendix B
T vs K (st, $50 \mathrm{~km} / \mathrm{hr}, \mathrm{s}+\mathrm{s} / \mathrm{km}>3$ )


Spiess (1990) develop the conical volume delay function formulation as a viable alternative to the widely use BPR (Bureau of Public Roads) function. This conical function ensures compatibility with the BPR and overcome some "inherent drawbacks" of this particular formulation.

The volume delay functions are usually expressed as the product of the free flow travel time (to) multiplied by a normalized congestion function $(f(x))$.

$$
t(v)=t o \cdot f\left(\frac{v}{c}\right)
$$

The BPR functions are define as:

$$
t^{B P R}(v)=t o \cdot\left(1+\left(\frac{v}{c}\right)^{\alpha}\right)
$$

$t o=$ free flow travel time
$v=$ volume
$c=$ critical flow
$\alpha=$ parameter to determine different shapes on the curves
notice that the congestion function is:
$f^{B P R}(x)=1+x^{\alpha}$
where $x=\frac{v}{c}$

BPR functions are shown in Figure 3.1 (pg. 58)

Spiess identified following "inherent drawbacks" of the BPR formulation:
a) With high values of $\alpha$ and volume critical flow ratios $(v / c)$ greater than 1, the BPR volume delay function travel times became too high. This usually occurs during the first few iterations of an equilibrium assignment. The extreme high travel times slow down convergence by giving undue weight to overloaded links with high $\alpha$ values and also can cause numerical problems such as overflow conditions and loss of precision.
b) For links use far below the critical flow and especially when high values of alpha are used, the BPR function yields free flow times independent of actual traffic volumes. Therefore the equilibrium model will locally degenerate to an all-or-nothing assignment, where the slightest change (or error) in free flow travel time may result in a complete shift of volume from one path to another path.
c) The evaluation of the BPR functions requires the computation of two transcendental functions, i.e. a logarithm and an exponential function to implement the power of $(v / c)^{\wedge}$ $\alpha$. This requires a fair amount of computing resources.

In order to overcome the BPR functions drawbacks (conditions 5,6 and 7, below) and ensure compatibility (conditions 1 to 4 , below), Spiess set the following requirements for a well-behaved congestion function:

1) $f(x)$ should be strictly increasing. This is a necessary condition for the assignment to converge to a unique solution.
2) $f(0)=1$ and $f(1)=2$. These conditions ensure compatibility with the BPR functions.
3) f '(x) should exist and be strictly increasing. This ensures convexity of the congestion function.
4) $f^{\prime}(1)=\alpha$. Alpha is, by analogy to the exponent in the BPR functions, the parameter that defines how the congestion effects change when the capacity is reached.
5) $f^{\prime}(x)<M \alpha$, where $M$ is a finite positive constant. The steepness of the congestion curve is thus limited. This prevents the values of the volume delay function from becoming too high when considering $v / c$ ratios higher than 1 .
6) $\mathrm{f}^{\prime}(0)>0$ This condition guarantees the uniqueness of the link volumes.
7) The evaluation of $f(x)$ should not take more computing time than does the evaluation of the corresponding BPR function.

The conical congestion function proposed met these requirements and is define as:

$$
f^{c}(x)=2+\sqrt{\alpha^{2}(1-x)^{2}+\beta^{2}}-\alpha(1-x)-\beta
$$

where $\beta$ is given as
$\beta=\frac{2 \alpha-1}{2 \alpha-2}$
$\alpha=$ parameter for congestion effect

Conical functions are shown in Figure 3.2 (pg. 59)

